

Tunable axial potentials for atom chip waveguides

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We present a method for generating precise magnetic potentials that can be described by a polynomial series along the axis of a cold atom waveguide near the surface of an atom chip. With a single chip design consisting of several wire pairs, various axial potentials can be created by varying the ratio of the currents in the wires, including double wells, triple wells, and pure harmonic traps with suppression of higher order terms. We use this method to design and fabricate a chip with modest experimental requirements. Finally, we use the chip to demonstrate a double well potential.

I. INTRODUCTION

Experiments with cold atoms often rely on carefully designed magnetic fields to create potentials for specific experimental requirements. One method for creating magnetic potentials are “atom chips,” where planar multi-layer structures with complex wire patterns are produced lithographically [1–3] or by etching direct bonded copper (DBC) on an aluminum nitride (AlN) substrate [4]. The general trapping parameters are determined by the wire pattern and the specific trapping potential is determined by the chip currents which enables a broad range of possible magnetic trapping parameters [5]. The magnetic traps can further be expanded with the addition of radio frequency [6] and microwave [7] fields. The extensive configurability and compact size of atom chips has made them a cornerstone of emerging atomic sensor technologies [8–11]. Precise control over the atom chip potential is becoming increasingly important, however, few designs take full advantage of the arbitrary configurability afforded by atom chips.

In this paper, we present a method for designing atom chips which produce tunable 1D magnetic potentials by summing the magnetic field contributions from multiple wire pairs. Both even and odd contributions are accessible, enabling arbitrary polynomial potentials to be realized. The linear combination of the fields produces a total magnetic field that can be written as a n th-order polynomial. We show that the order of the polynomial is determined by the number and locations of the wire pairs and that the coefficients of the polynomial are determined by the ratio of currents in the wire pairs. Thus, a single chip design can be used to realize a wide range of trapping potentials. This tunability applies to a variety of experiments, including trapped atom interferometry [12, 13], chip-based precision measurements [14], 1D

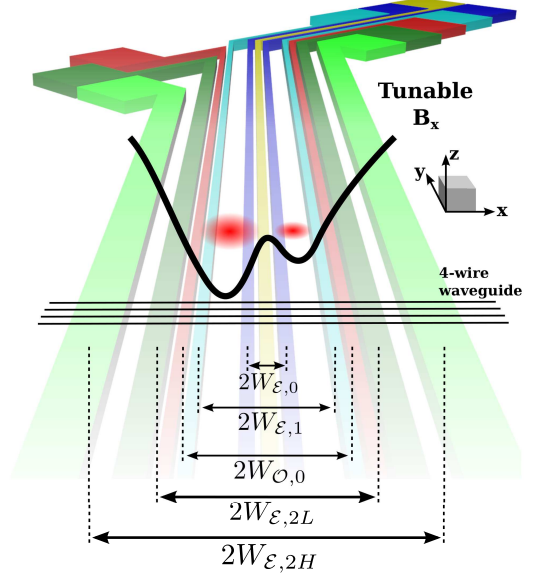


FIG. 1. Multilayer atom chip with tunable control over B_x along a cold atom waveguide. The black crossing wires on the top layer are used to form a four-wire waveguide. Below, even and odd wire pairs, spaced by $2W_{P,m}$ give control over even and odd contributions to the 1D potential.

Bose gases [15], and atomtronic devices [5, 10, 16].

In practice, canceling higher order terms requires a higher power dissipation. Therefore, we consider low power wire configurations that relax the requirements on higher-order terms. Based on the calculations outlined in this paper, we have designed, fabricated, and tested an atom chip capable of controlling the 1D potential including both the optimal and reduced power wire configurations.

The rest of the paper is organized as follows. In Sec. II we present an idealized atom chip and its corresponding 1D polynomial potential, and examine the tunability of both even and odd terms. In Sec. III, we describe an

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example chip design and solve for wire currents using either the optimal or the low power configuration and in Sec. IV we present initial experimental results showing the tunability of the potential and summarize our findings.

II. MAGNETIC FIELD CONTROL IN 1D

In many experiments, an atom cloud is sufficiently confined in two directions such that its dynamics can be described by a one-dimensional (1D) equation of motion. In this paper, a radial plus an effective 1D axial potential is formed by pairs of wires patterned on a two layer atom chip with an adjustable uniform external magnetic field. This chip is shown schematically in Fig. 1. The layer closest to the atoms will be used to create a magnetic waveguide [17], depicted as a set of four horizontal black wires, that tightly confines the atoms in the radial directions. The far layer is composed of multiple wire pairs which create a tunable axial field perpendicular to the waveguide. The wire pairs will be referred to as pinch wires, since they act much like the pinch coils in a Ioffe trap. While Fig. 1 shows finite wires with leads, the following derivation assumes infinitely long thin wires.

In Appendix A, we show the axial and radial potentials are separable when $\mu|B_x(0)| \gg m\omega_\perp^2 \sigma_\perp^2$, where σ_\perp is the characteristic size of the atomic cloud in the radial direction, and $B_x(0)$ represents the total bottom field in the waveguide. The effective 1D axial potential along x can then be written as

$$V = \mu|B_x(x)| + \frac{1}{2}m\omega_\perp^2 r_\perp^2, \quad (1)$$

where $\mu = \mu_B g_F m_F$ is the magnetic moment of the atomic state that is trapped, μ_B is the Bohr magneton, g_F the the Lande g-factor, m_F is the magnetic quantum number, $B_x(x)$ is the tunable magnetic field in the \hat{e}_x -direction, ω_\perp is the trapping frequency in the radial direction, r_\perp is the distance from the trap axis, and m is the atomic mass.

For a single wire pair centered about the origin with both currents running in the \hat{e}_y -direction the field can be expressed as the following series

$$B_x(x) = \frac{\mu_0 I}{2\pi H} \left[c^{(0)} + c^{(2)} \left(\frac{x}{H} \right)^2 + c^{(4)} \left(\frac{x}{H} \right)^4 + \dots \right], \quad (2)$$

where I is the current in the wire pair and H is the distance of the trap from the plane of the wires. In Appendix B, we show that the parameters $c^{(n)}$ are given by the relation

$$c^{(n)}(w) = \frac{2}{(1+w^2)^{n+1}} \sum_{r=0}^n (-1)^{(n+r)/2} \binom{n+1}{r} w^r \phi_{n+r}, \quad (3)$$

for any n , where $\phi_a = (1 + (-1)^a)/2$ is a parity function of the integer argument a that is one if a is even and zero

if a is odd. Fig. 2 shows the first few even values of the coefficients $c^{(n)}$, given in Eq. (3), as a function of half the wire spacing $w_\mathcal{E} = W_\mathcal{E}/H$ where $W_\mathcal{E}$ is the half wire spacing between the wire pairs. The odd orders cancel due to the symmetry of the wire spacing and the currents. For anti-symmetric current flow there is series similar to Eq. (2) except the even orders cancel such that there are only odd terms. Equation (3) holds for both even and odd contributions to the potential.

Akin to Helmholtz or anti-Helmholtz coil pair there is a particular wire spacing where one of the orders in the expansion will cancel. Changing the current of a single wire pair will equally scale all of the orders in the series expansion but does not change the functional form of the potential. We use multiple wire pairs at various spacings such that the potential is a linear combination of the series expansions of each wire pair. Individual terms in the total tunable field can then varied by changing the relative currents in the wire pairs.

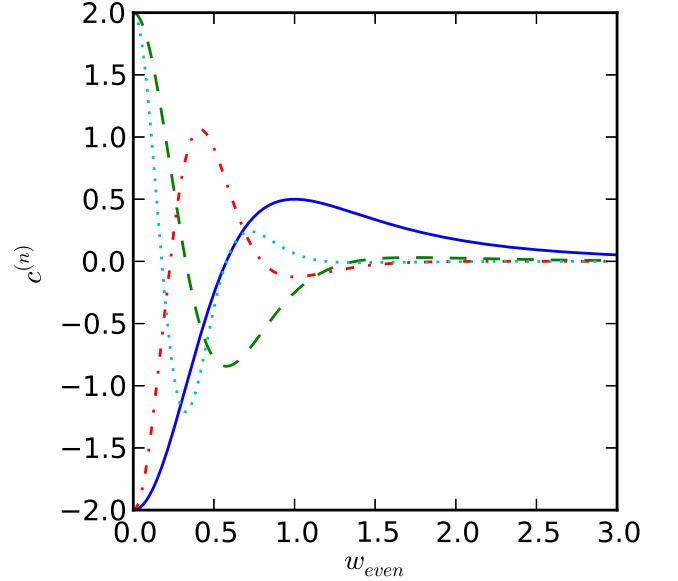


FIG. 2. The lowest few even coefficients, $c^{(n)}$ are shown. The solid line is $c^{(2)}$, the dashed line is $c^{(4)}$, the dash-dot line is $c^{(6)}$, and the dotted line is $c^{(8)}$.

The total tunable field can be expanded into the following series (N.B. the c 's denote the magnetic field contribution from a single wire and the C 's are a sum of c 's)

$$B_x(x) = B_x^* + B_R \left[C^{(0)} + C^{(1)} \left(\frac{x}{H} \right) + C^{(2)} \left(\frac{x}{H} \right)^2 + C^{(3)} \left(\frac{x}{H} \right)^3 + C^{(4)} \left(\frac{x}{H} \right)^4 + \dots \right], \quad (4)$$

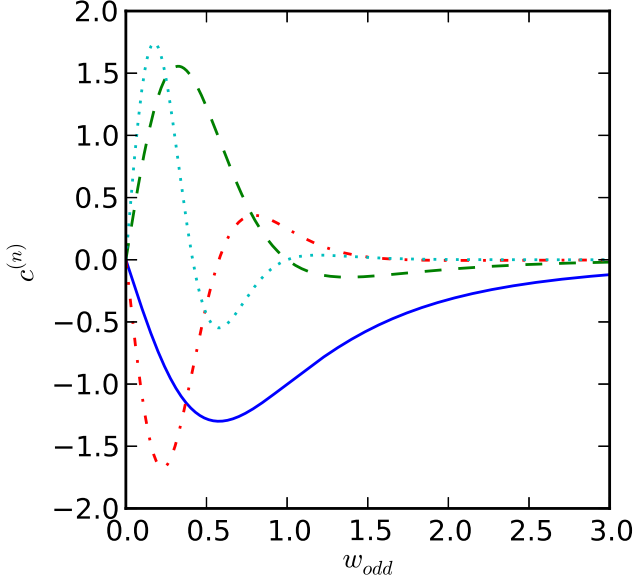


FIG. 3. The lowest few odd coefficients, $c^{(n)}$ are shown. The solid line is $c^{(1)}$, the dashed line is $c^{(3)}$, the dash-dot line is $c^{(5)}$, and the dotted line is $c^{(7)}$.

where

$$C^{(n)} = \sum_{m=0}^{M(\mathcal{P})-1} i_{\mathcal{P},m} c^{(n)}(w_{\mathcal{P},m}). \quad (5)$$

where $i_{\mathcal{P},m} = I_{\mathcal{P},m}/I_R$ is the relative current in the m -th wire pair, $w_{\mathcal{P},m} = W_{\mathcal{P},m}/H$, scaled by the distance from the waveguide axis to the wire plane. The parity of n determines which wire pairs contribute to $C^{(n)}$. The number of contributing wire pairs is given by $M(\mathcal{P})$, where \mathcal{P} denotes the parity of n , denoted either \mathcal{E} for even, or \mathcal{O} for odd. Additionally, B_x^* is the magnitude of the externally-applied, uniform bias field in the x -direction, B_R is the overall potential scaling given by

$$B_R = \frac{\mu_0 I_R}{2\pi H}, \quad (6)$$

where I_R is the reference current.

The rest of this section describes how arbitrary values of the $C^{(n)}$ coefficients can be generated from a particular wire configuration.

To ensure separability we compute the bottom field by summing the externally applied field with terms from the even wire pairs,

$$B_x(0) = B_x^* + B_R \sum_{m=0}^{M(\mathcal{E})-1} i_{\mathcal{E},m} c^{(0)}(w_{\mathcal{E},m}). \quad (7)$$

Note that the odd wires make no contribution to the bottom field.

The magnetic field of the wire pairs also consists of a component in the z -direction. This field can be expanded

as

$$B_z(x) = B_z^* + B_R \left(D^{(0)} + D^{(1)} \frac{x}{H} + \dots \right). \quad (8)$$

The opposite parity condition in the z -direction means that the dimensionless parameters, $D^{(n)}$, are determined by the currents in wires of parity $n+1$, of which there are $M(\mathcal{P}')$. These coefficients are given by,

$$D^{(n)} = \sum_{m=0}^{M(\mathcal{P}')-1} i_{\mathcal{P}',m} d^{(n)}(w_{\mathcal{P}',m}), \quad (9)$$

where $d^{(n)}$ are dimensionless parameters that depend only on the spacing of the wires.

The parameter $D^{(0)}$ causes a displacement of the waveguide in the z -direction. However, this constant field can be corrected for with the addition of a uniform bias field, B_z^* . In the rest of the paper we assume that the correct bias field is applied.

Non-zero values of $D^{(1)}$ cause a rotation of the waveguide. Typically, this rotation is set to zero, however there are situations where changing this rotation angle will be useful, such as the alignment of a cloud with a standing wave laser field. Extensions to non-zero rotations are straightforward, but will be neglected in what follows.

A general expression for $d^{(n)}$, similar to the one found in Appendix B, may be found. However, we are only interested in the two lowest orders which can be expressed as

$$d^{(0)}(w_{\mathcal{O},m}) = 2 \frac{w_{\mathcal{O},m}}{w_{\mathcal{O},m}^2 + 1}, \quad (10)$$

and

$$d^{(1)}(w_{\mathcal{E},m}) = -2 \frac{(w_{\mathcal{E},m} - 1)(w_{\mathcal{E},m} + 1)}{(w_{\mathcal{E},m}^2 + 1)^2}. \quad (11)$$

The currents in a set of $M(\mathcal{E})$ wire pairs can be used to control, usually the lowest $M(\mathcal{E}) - 1$ terms from Eq. (5), plus the parameter $D^{(1)}$, from Eq. (9). For a given set of wire spacings, $\{w_{\mathcal{E}}\}$, the currents can be found by inverting Eqns. (5) and (9). Once the currents have been found, the contributions to the potential from the uncontrolled parameters can be calculated.

By placing wire pairs at the roots of a coefficient, we can eliminate the contribution to the potential from that coefficient. Fig. 2 shows the first few even values of the coefficients $c^{(n)}$. Each even coefficient has one more zero crossing than the previous one, i.e. $c^{(2)}$ has one root, $c^{(4)}$ has two roots, $c^{(6)}$ has three roots, etc. Thus, an atom chip can be designed to produce a polynomial of any even order with the next highest order being exactly canceled.

The number of roots is exactly the number of wires needed to control all of the lower coefficients plus $D^{(1)}$. By placing wire pairs at all of the roots of a given even coefficient, and controlling the relative current through each pair, one can tune the lower even coefficients, as well as the additional coefficient $D^{(1)}$. For example, by

placing wires at the three roots of $c^{(6)}$, we can independently control the three parameters $C^{(2)}$, $C^{(4)}$, and $D^{(1)}$, while also having $C^{(6)} = 0$.

Similarly, the currents in a set of $M(\mathcal{O})$ wire pairs can be used to control $M(\mathcal{O})$ terms from Eq. (5). Once the wire spacing's $\{w_{\mathcal{O}}\}$ have been determined the currents are found by inverting Eq. (5) and (9). An applied bias field of $B_z^* = -B_R D^{(0)}$ is required to cancel the $D^{(0)}$ that arises from the odd wire pairs. The value can be calculated from the currents, and using Eq. (9) and (10).

Fig. 3 shows the first few odd values of the coefficients $c^{(n)}$, given in Eq. (3), as a function of half the wires spacing $w_{\mathcal{O}} = W_{\mathcal{O}}/H$. The solid line is $c^{(1)}$, the dashed line is $c^{(3)}$, the dash-dot line is $c^{(5)}$, and the dotted line is $c^{(7)}$. Like with the even case, each of the odd coefficients has one more root than the previous one. However, one of the roots is always at $w_{\mathcal{O}} = 0$. This root cannot be used to create an odd potential, and is therefore not useful. As a result, $c^{(1)}$ has no useful roots, $c^{(3)}$ has one useful root, $c^{(5)}$ has two useful roots, etc.

By placing the odd wires at one of the useful roots of a coefficient, all of the lower coefficients can be controlled. For example, by placing wires at the two roots of $c^{(5)}$, the coefficients $C^{(1)}$ and $C^{(3)}$ can be controlled, and $C^{(5)} = 0$. The dominant contribution of the z -component of the field, $D^{(0)}$, can be eliminated using a bias field. It is not necessary to have a wire pair to control its value.

III. EXAMPLES

We will first determine the placement of the wire pairs and then describe two potentials that can be generated with the design. Consider the case of three even wire pairs and two odd wire pairs. These wires can be used to create any potential that is described by a fourth order polynomial. Once the coefficients and wire spacing's are specified, the set of currents, $\{i_{\mathcal{P}}\}$, can be found by solving the following matrix equations. For the even wires,

$$\begin{pmatrix} c^{(2)}(w_{\mathcal{E},0}) & c^{(2)}(w_{\mathcal{E},1}) & c^{(2)}(w_{\mathcal{E},2}) \\ c^{(4)}(w_{\mathcal{E},0}) & c^{(4)}(w_{\mathcal{E},1}) & c^{(4)}(w_{\mathcal{E},2}) \\ d^{(1)}(w_{\mathcal{E},0}) & d^{(1)}(w_{\mathcal{E},1}) & d^{(1)}(w_{\mathcal{E},2}) \end{pmatrix} \begin{pmatrix} i_{\mathcal{E},0} \\ i_{\mathcal{E},1} \\ i_{\mathcal{E},2} \end{pmatrix} = \begin{pmatrix} C^{(2)} \\ C^{(4)} \\ 0 \end{pmatrix}, \quad (12)$$

and for odd wires,

$$\begin{pmatrix} c^{(1)}(w_{\mathcal{O},0}) & c^{(1)}(w_{\mathcal{O},1}) \\ c^{(3)}(w_{\mathcal{O},0}) & c^{(3)}(w_{\mathcal{O},1}) \end{pmatrix} \begin{pmatrix} i_{\mathcal{O},0} \\ i_{\mathcal{O},1} \end{pmatrix} = \begin{pmatrix} C^{(1)} \\ C^{(3)} \end{pmatrix}. \quad (13)$$

Eqns. (12) and (13) can be used to set the coefficients $C^{(1)}$ through $C^{(4)}$ for any given wire spacing. However, contributions to the higher order terms of the potential will generally depend on these wire spacings.

The sixth order contribution can be eliminated, $C^{(6)} = 0$ by placing the wires with spacing of $w_{\mathcal{E},0} = 0.228$,

$w_{\mathcal{E},1} = 0.797$, and $w_{\mathcal{E},2H} = 2.076$. The fifth order contribution is always zero, $C^{(5)} = 0$, when $w_{\mathcal{O},0} = 0.577$, and $w_{\mathcal{O},1} = 1.732$.

In situations where small sixth order contributions to the potential can be tolerated, the total power consumption of the atom chip can be greatly reduced by moving the outer pair of wires closer together. We choose to place the outer wires at a spacing where $w_{\mathcal{E},2} = w_{\mathcal{E},2L} = 1.3$. This choice has a much lower power requirements than the optimal spacing while maintaining a rather low contribution from the sixth order term.

Several example trap configurations will now be discussed. There will be four free parameters $C^{(1)}$ through $C^{(4)}$. For each trap type, the results will be presented for the optimal configuration where $w_{\mathcal{E},2} = w_{\mathcal{E},2H} = 2.076$, and a low power configuration where $w_{\mathcal{E},2} = w_{\mathcal{E},2L} = 1.3$. For both of these configurations, it will be assumed that the odd wire spacing's are in the optimal configuration discussed above.

For the optimal and low power configurations, Eqns. (12) and (13) are numerically inverted. For the case of the optimal configuration the currents are given by the relations

$$\begin{pmatrix} i_{\mathcal{E},0} \\ i_{\mathcal{E},1} \\ i_{\mathcal{E},2H} \end{pmatrix} = \begin{pmatrix} 1.33 & 1.47 & 1.05 \\ 2.31 & 0.70 & 1.64 \\ 12.37 & 11.54 & 5.31 \end{pmatrix} \begin{pmatrix} C^{(2)} \\ C^{(4)} \\ D^{(1)} \end{pmatrix} \quad (14)$$

and for the low power configuration

$$\begin{pmatrix} i_{\mathcal{E},0} \\ i_{\mathcal{E},1} \\ i_{\mathcal{E},2L} \end{pmatrix} = \begin{pmatrix} 0.31 & 0.52 & 0.61 \\ 0.29 & -1.18 & 0.77 \\ 3.18 & 2.96 & 1.36 \end{pmatrix} \begin{pmatrix} C^{(2)} \\ C^{(4)} \\ D^{(1)} \end{pmatrix}. \quad (15)$$

With one exception the magnitude of the currents in the high power configuration Eq. (14) are always larger than the values in the low power configuration. This is especially true of the last row in the matrices, which determines the current in the outer most wire.

Assuming the resistance of each of the pinch wires are equal, the total power dissipated is given as the sum of the squares of the currents. For the harmonic potential in the optimal configuration, the power dissipation is proportional to $\sum_m i_m^2 = 160.00$ and in the low power configuration the power dissipation is proportional to $\sum_m i_m^2 = 10.27$. For the case of a harmonic potential, the power dissipation due to the pinch wires is 15 times less for the low power configuration. In addition, the low power configuration requires a smaller external bias. We are interested in the case where $D^{(1)} = 0$, so the last row in both matrices will not be used in the discussion that follows below.

The inverted equation for the odd terms is

$$\begin{pmatrix} i_{\mathcal{O},0} \\ i_{\mathcal{O},1} \end{pmatrix} = \begin{pmatrix} -0.19 & 0.77 \\ -1.73 & -2.31 \end{pmatrix} \begin{pmatrix} C^{(1)} \\ C^{(3)} \end{pmatrix}. \quad (16)$$

Finally, the bias field needed to cancel the $D^{(0)}$ term, is

$$B_z^*/B_R = 1.66C^{(1)} + 1.33C^{(3)}. \quad (17)$$

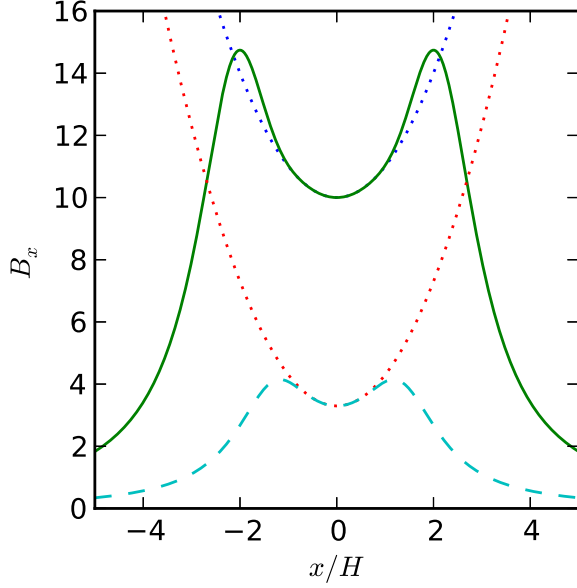


FIG. 4. With $C^{(2)} = 1$, and all higher order terms zeroed, the field along x is harmonic. The solid (green) curve shows the field produced by the wires in the optimal configuration $w_{\mathcal{E},2} = w_{\mathcal{E},2H}$. The dashed (cyan) curve shows the field produced by the wires in the low power configuration $w_{\mathcal{E},2} = w_{\mathcal{E},2L}$. Dotted lines represent idealized harmonic field profiles.

A. Harmonic trap

This tunable trap will be useful for atom interferometry in harmonic traps. This is particularly true for an interferometer that uses trapped thermal atoms, because contributions to the fourth (and higher) order term cause decoherence due to the larger size of cloud which samples more of the potential. Additionally, higher order contributions to the potential can be caused by the finite length of chip wires, the leads that connect the chip wires to the power supplies, ion pumps, or other laboratory equipment. These contributions can be canceled by tuning the parameter $C^{(4)}$, while holding $C^{(2)}$ constant. The $C^{(4)}$ can be tuned both positive and negative to cancel any stray $C^{(4)}$ coefficient. To effectively remove the effects of the fourth order contributions to the potential, it must first be determined. We are currently developing methods for measuring these fourth order contributions and plan on using the chip described in this paper to evaluate the effectiveness of these methods.

Before tuning the parameter $C^{(4)}$, a harmonic trap must first be created and loaded. Fig. 4 shows the magnetic field for the case where $C^{(2)} = 1$, and all other coefficients are zero. The solid (green) curve shows the field produced by the wires in the optimal configuration $w_{\mathcal{E},2} = w_{\mathcal{E},2H}$, and the dashed (cyan) curve shows the field produced by the wires in the low power configuration $w_{\mathcal{E},2} = w_{\mathcal{E},2L}$. The dotted lines show the field profile when higher order terms are neglected. With the

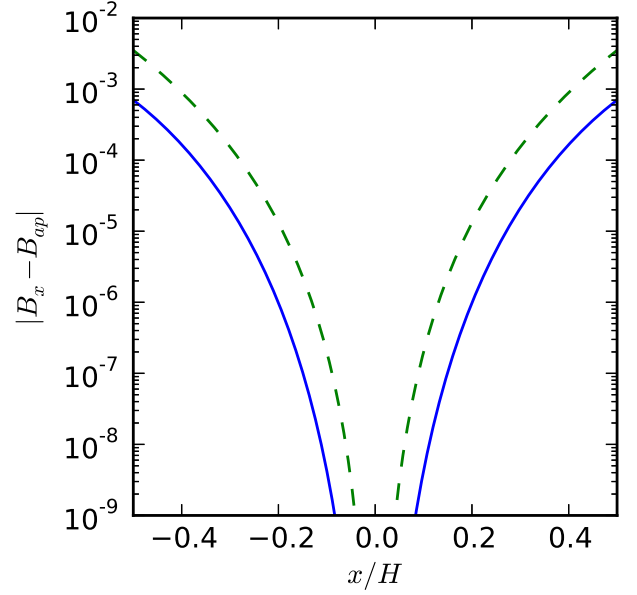


FIG. 5. Comparison of the deviations of the axial field from pure harmonicity for optimal configuration versus the low power configuration. The high power configuration (solid blue) results in an order of magnitude improvement over the low power configuration (dashed green).

pinch wires in the optimal configuration, the trap remains harmonic over a larger range. The optimal trap is also deeper and has a larger bottom field. Thus, the bias field to reduce the bottom field will need to be larger for the optimal configuration as compared to the low power configuration.

To quantify the effects of the uncontrolled higher order contributions of the field, Fig. 5 shows a log plot of the difference between the simulated field keeping higher order terms, and an ideal parabola given by $B_{ap} = C^{(0)} + x^2$, where $C^{(0)}$ is found using Eq. (5) for the two wires spacing's. The solid (blue) curve shows the difference in the optimal wire configuration and the dashed (green) curve shows the difference in the low power configuration. The low power configuration produces a field that is about an order of magnitude “less harmonic” than the wires in the optimal configuration. However, for sufficiently small atomic clouds, $\sigma_{\parallel}/H < 0.05$, where σ_{\parallel} is the axial size of the atomic cloud, both configurations produce potentials that are harmonic to one part in 10^{-9} .

To determine the amount of current that needs to be run in each wire of the chip, we need to determine the scaling of the current. To make a harmonic trap, with trap frequency ω , the scaling current should be

$$I_R = \frac{\pi H^3 m \omega^2}{\mu \mu_0}, \quad (18)$$

where as before μ is the magnetic moment of the trapped state. Trapping ^{87}Rb in the $F = 2$, $m_f = 2$ state, in a trap with frequency $\omega = 2\pi \times 10$ Hz that is $H = 2$ mm

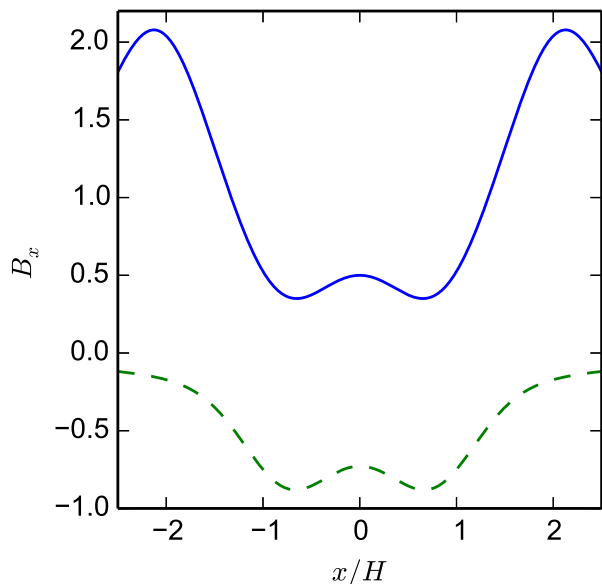


FIG. 6. A double well potential created by tuning the terms $C^{(2)}$ and $C^{(4)}$. For both configurations $C^{(2)} = -0.75$ and $C^{(4)} = 1.0$. In the optimal configuration (solid blue) the potential is deeper and has a large bottom field that may be offset with an external bias field. In the low power configuration (dashed green) the field is negative and would need to be offset with an external bias field to maintain the shape of the double well.

from the pinch wires, means that $I_R = 1.22$ A. Applying this scaling current, the currents in the high power configuration are $I_{\mathcal{E},0} = 1.62$ A, $I_{\mathcal{E},1} = 2.81$ A, $I_{\mathcal{E},2H} = 15.08$ A, with a bottom field of $B_R C^{(0)} = 12.20$ Gauss. For the low power configuration the currents are $I_{\mathcal{E},0} = 0.37$ A, $I_{\mathcal{E},1} = 0.34$ A, $I_{\mathcal{E},2L} = 3.87$ A, and the bottom field is $B_R C^{(0)} = 4.03$ Gauss.

B. Double well trap

The same chip can be used to produce a double well trap, where both the distance between the two traps and the difference between the potential at the bottom of each trap can be independently tuned. This type of double well trap can be used to study the merging of two cold or ultra-cold atomic clouds, the quantum dynamics of a BEC in a double well potential or most interestingly it may be useful as a coherent splitter for a BEC.

Fig. 6 shows a double well magnetic field produced by our chip. The solid (green) curve in Fig. 6 shows the magnetic field produced by the pinch wires in the optimal configuration for a double well trap with parameters $C^{(2)} = -0.75$, and $C^{(4)} = 1$. The dashed (cyan) curve is the field produced by the pinch wires in the low power configuration. The two dotted curves are the approximate values when no higher order contributions to the field are included. This figure is an example of how two

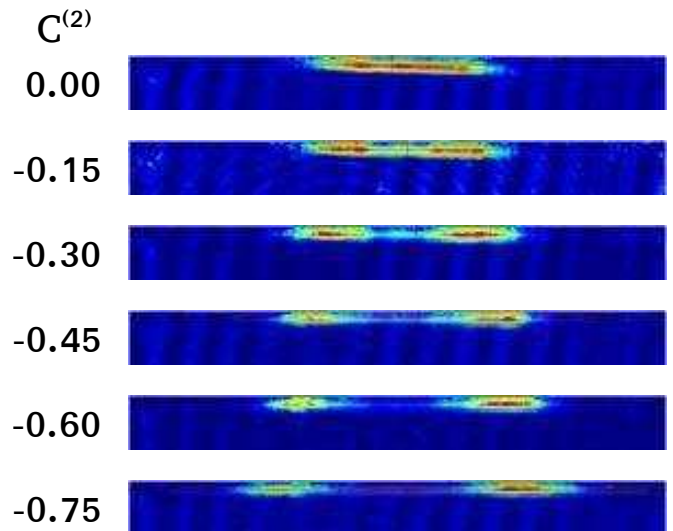


FIG. 7. Experimental images of an atom cloud at appropriately $2 \mu\text{K}$ being transitioned from a pure $C^{(4)} = 1$ state to the double well state shown in Fig. 6 as a function of the $C^{(2)}$ parameter. There is an apparent tilt between the two wells that results in a number imbalance between the two wells. It has not been determined if this is a physical tilt of the waveguide or a \hat{e}_z gradient.

traps that have the same shape near the origin can have very different behavior far from the origin. For the trap created using the wires in the optimal configuration, the bottom field is positive. To reduce the size of this bottom field, a negative bias field must be applied. The field has a maximum before it tends towards zero. On the other hand, for the low power configuration, the field is always negative. To create a double well trap at all, there must be a positive bias field applied to lift the field such that it is always positive. The field has no other extrema, and tends towards zero after the double well structure.

For this particular choice of parameters $C^{(2)}$ and $C^{(4)}$, the locations of the two wells should be $x/H = \pm\sqrt{3/8}$. Fig. 6 shows that the locations of the wells is slightly larger than this at $x/H \sim \pm 0.66$. To create a trap with frequency of 10 Hz, the reference current should be $I_R = 0.81$. For the high power configuration the currents should be $I_{\mathcal{E},0} = 0.38$ A, $I_{\mathcal{E},1} = -0.82$ A, and $I_{\mathcal{E},2H} = 1.83$ A. The magnetic field at the trap minimum is 0.28 Gauss. For the low power configuration, $I_{\mathcal{E},0} = 0.23$ A, $I_{\mathcal{E},1} = -1.13$ A, and $I_{\mathcal{E},2L} = 0.473$ A. The magnetic field at the trap minimum is -0.71 Gauss.

We show preliminary experimental results of a tunable atom chip well in Fig. 7 where an approximately $2 \mu\text{K}$ atom cloud of ^{87}Rb atoms in the $|F=2, m_F=2\rangle$ state are trapped on an atom chip similar to the chip shown in Fig. 1. The pure 4th order potential is modified by the addition of a negative $C^{(2)}$ contribution that splits the potential into two wells. Further results are being prepared for future publications.

IV. CONCLUSIONS

We have demonstrated that tunability of an axial magnetic field in a cold atom waveguide can be achieved with sets of paired wires on an atom chip. By symmetry, wires with (anti-) parallel currents only contribute to the (odd) even terms in the polynomial expansion of the field along the guide axis. When a wire pair is placed at a zero of a particular coefficient, it allows the lower order terms (of the same parity) to be adjusted without contributing to the coefficient itself. Several wire pairs, appropriately placed, lead to arbitrary tunability of $N - 1$ coefficients simply by controlling the relative currents through the sets of wire pairs. Experiments that employ 1D potentials now have a tool with which precise potentials may be generated from a double layer atom chip. We have also shown the initial operation of a tunable atom chip by trapping a $2 \mu\text{K}$ cloud of ^{87}Rb atoms trapped in a pure 4th order potential and in a double well configuration that is comprised of $C^{(2)} = -0.75$ and $C^{(4)} = 1.0$.

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Appendix A: WAVEGUIDE PLUS AXIAL FIELD

A magnetic waveguide is a field configuration where the magnetic field vanishes along an axis. Near the zero, the field points perpendicular to the guide and can be described by a single parameter G , which is the magnetic field gradient of the waveguide. For example, the magnetic field for a waveguide that points in the x -direction can be written as

$$B_{\text{radial}} = G(\hat{e}_y y - \hat{e}_z z). \quad (\text{A1})$$

The 1D potential will be created using a magnetic field that is produced by the current in several wires that run parallel to the y -axis (perpendicular to the waveguide axis). This field will provide confinement in the axial direction and will be assumed to be of the form,

$$B_{\text{axial}} = B_x(x, z)\hat{e}_x + B_z(x, z)\hat{e}_z. \quad (\text{A2})$$

The z -dependence in Eq. (A2) causes two small shifts to the potential. First, it causes a change in the gradient in the z -direction, which can be neglected when $G \gg \frac{\partial B_x}{\partial z}$. Next, it causes a displacement in the z -direction, which can be neglected when $G^2 \sigma_x \gg \frac{\partial B_x^2}{\partial z}$ where σ_x is the size of the cloud in the x -direction. When these inequalities are satisfied, the Eq. (A2) reduces to

$$B_{\text{axial}} = B_x^T(x)\hat{e}_x + B_z(x)\hat{e}_z. \quad (\text{A3})$$

The x -component of the magnetic field creates the potential along the waveguide, and is the field that we

wish to control. The z -component of this field causes deformations to the waveguide. The constant term, $B_z^{(0)} = B_R D^{(0)}$ in Eq. (8), causes a shift in the location of the guide by $B_z^{(0)}/G$ along the y -axis, which can be corrected using a uniform bias field in the z -direction. Let us assume the appropriate zeroing bias is applied. The second term causes a rotation of the waveguide about z in the $x - y$ plane. The waveguide is rotated by the angle, $\theta \approx B_z^{(1)}/G$. When using optical pulses to manipulate the state of the trapped atoms, this rotation angle becomes important. Typically, this angle will be set to zero, and neglected. However, including nonzero rotations is straightforward. The higher order contributions to B_z cause other distortions to the path of the waveguide, but below those effects will not be considered.

With B_z set to zero, the field along the waveguide axis can be separated into two parts: a non-zero “bottom field”, $B_x(0)$ which prevents spin-flip losses and is necessary for the potential to be separable, and $B_x(x)$, the part of the axial field that depends on the x -coordinate,

$$B_{\text{axial}} = B_x^T(x) = B_x(0) + B_x(x). \quad (\text{A4})$$

The potential that the atoms experience is obtained from the radial and axial components, given by Eq. (A1) and Eq. (A4) respectively, as follows

$$V = \mu \sqrt{(B_x(0) + B_x(x))^2 + G^2 r_{\perp}^2}, \quad (\text{A5})$$

where μ is the magnetic moment of the trapped state and $r_{\perp} = \sqrt{y^2 + z^2}$ is the radial coordinate.

Assuming that $B_x(0) \gg B_x(x)$ and expanding Eq. (A5) yields

$$V = \mu \left(|B_x(0) + B_x(x)| + \frac{1}{2} \frac{G^2}{|B_x(0)|} r_{\perp}^2 - \frac{1}{2} \frac{G^2}{|B_x(0)| B_x(0)} B_x(x) r_{\perp}^2 \right). \quad (\text{A6})$$

The last term in Eq. (A6) is clearly not separable, i.e. it cannot be written in the form $V = V_{\text{axial}}(x) + V_{\text{radial}}(r_{\perp})$. However, the potential may be regarded as separable in the limit where $B_x(0)^2 \gg G^2 \sigma_{\perp}^2$, where σ_{\perp} is the size of the atomic cloud in the radial direction.

From Eq. (A6) it is clear that the potential along the waveguide can be written in the form shown in Eq. (1).

Appendix B: The general expression

Take the surface of the atom chip to be at $z = 0$, with an infinitely long wire parallel to the y -axis along the line $x = W$. When a current of I_R is passed through the wire, the x -component of a magnetic field a distance H above the field point X is given by,

$$B_x(x) = B_R \frac{1}{1 + (x' - w)^2}, \quad (\text{B1})$$

where $x' = X/H$, and $w = W/H$, and $B_R = \mu_0 I_R / 2\pi H$. Eq. (B1) can be expanded as the series

$$B_x(x)/B_R = \sum_{n=0}^{\infty} \sum_{k=0}^{2n} (-1)^{(n+k)} \binom{2n}{k} w^{2n-k} x^k, \quad (\text{B2})$$

where the primes have been dropped. To determine the coefficients for each power of x , the order of the summation in Eq. (B2) needs to be interchanged. To do this first, the even and odd terms are separated so that the upper limit of the inner summation can be divided in half, i.e. $\sum_{k=0}^{2n} A_{n,k} = \sum_{k=0}^n (A_{n,2k} + A_{n+1,2k+1})$. Then, the order summation can be flipped $\sum_{n=0}^{\infty} \sum_{k=0}^n B_{n,k} = \sum_{k=0}^{\infty} \sum_{n=k}^{\infty} B_{n,k} = \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} B_{k+n,k}$. Finally, after interchanging the order of the summation, Eq. (B2) becomes

$$B_x(x)/B_R = \sum_{k=0}^{\infty} (\alpha^{2k} x^{2k} + \alpha^{2k+1} x^{2k+1}), \quad (\text{B3})$$

where

$$\alpha^{(n)} = \sum_{q=0}^{\infty} (-1)^{(n+q)/2} \binom{n+q}{n} w^q \phi_{n+q}, \quad (\text{B4})$$

where $\phi_a = (1 + (-1)^a)/2$ is zero when a is odd and one when a is even.

Using the identity

$$\binom{n+q}{n} = \sum_{r=0}^{r_{max}} \binom{n+(q-r)/2}{n} \binom{n+1}{r} \phi_{q+r}, \quad (\text{B5})$$

where $r_{max} = \min(q, n+1)$, and it is assumed that both q and n are positive integers. Substituting (B5) into (3) and reversing the order of the summation yields

$$\alpha^{(n)} = \sum_{r=0}^{n+1} \sum_{q=r}^{\infty} (-1)^{(n+q)/2} \binom{n+(q-r)/2}{n} \binom{n+1}{r} \cdot w^q \phi_{q+r} \phi_{n+q}. \quad (\text{B6})$$

Eq. (B6) can be written as the product of two sums, by introducing the new index, $\kappa = (q-r)/2$ resulting in

$$\alpha^{(n)} = \left[\sum_r (-1)^{(n+r)/2} \binom{n+1}{r} w^r \phi_{n+r} \right] \cdot \left[\sum_{\kappa} (-1)^{\kappa} \binom{n+\kappa}{n} w^{2\kappa} \right]. \quad (\text{B7})$$

Recognizing that the second term in Eq. (B7) can be written as $(1+w^2)^{-n-1}$, we can write the coefficients as

$$\alpha^{(n)}(w) = \frac{1}{(1+w^2)^{n+1}} \sum_{r=0}^n (-1)^{(n+r)/2} \binom{n+1}{r} w^r \phi_{n+r}. \quad (\text{B8})$$

Each of the wires contribute to all of the coefficients. Contributions to the magnetic field with definite parity can be created using pairs of wires. A pair of wires will be located at $\pm w$. If the current is running in the same (opposite) direction, only even (odd) terms will contribute to the potential. For a pair of wires, the coefficients will be larger by a factor of two, i.e. $c^{(n)} = 2\alpha^{(n)}$.

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